J. Fluid Mech. (1975), *vol.* 72, *part* 2, *pp.* 341–352
Printed in Great Britain

On wavelike perturbations in a free jet travelling faster than the mean flow in the jet

By D. BECHERT AND E. PFIZENMAIER
DFVLR-Institut für Turbulenzforschung, 1 Berlin 12,
Müller–Breslau-Strasse 8

(Received 19 June 1974 and in revised form 27 May 1975)

The phase velocities of axial disturbances to a free jet have been investigated experimentally. For a certain range of low Strouhal numbers previous theories predicted the existence of phase velocities exceeding the mean velocity of the jet. These 'ultra-fast' phase velocities were found in the present experiments too.

1. Introduction

If one considers the fluctuations in a free jet, one does not expect the phase velocity of any perturbation to exceed the highest mean velocity in the flow field. It is difficult to understand why such an 'ultra-fast' perturbation phase velocity should occur, because the mean flow vorticity is convected at velocities certainly not exceeding the highest mean flow velocity. However, the differential equation for the stability of parallel inviscid flows can be derived from the vorticity transport equation; and the solution of this stability differential equation yields wave motions in a free jet with phase velocities exceeding the highest mean flow velocity occurring in the jet. The existence of these ultra-fast phase velocities will be demonstrated by the present experimental investigation.

Using a stability-theory approach, Michalke (1970) and Crow & Champagne (1971) predicted ultra-fast perturbation phase velocities exceeding the mean flow velocity by about 30% for a circular jet. The same type of ultra-fast wave motion occurs in plane jets for the case of symmetrical deformations of both jet boundaries. This can be concluded from the wavenumber behaviour calculated by Bechert (1972). The above-mentioned theoretical investigations assumed *spatial* amplification of the instability waves. It can be accepted as fact that it is correct to compare only *spatial* stability theory with experimental data obtained in a flow excited by sound of constant magnitude (as is the case in our experiments) because in both theory and experiment the frequency is real. The first calculations of spatially amplified waves in cylindrical jets (see Michalke 1970; Crow & Champagne 1971) were carried out under the assumption of an infinitesimally thin boundary layer around the circular jet. Consequently these theoretical results could hardly be compared with experimental data. Later on, in a more elaborate paper, Michalke (1971) took account of the finite thickness of the boundary layer. Thus the divergence of theoretical and experimental results in Crow & Champagne's paper turned out to be caused by the assumption of an

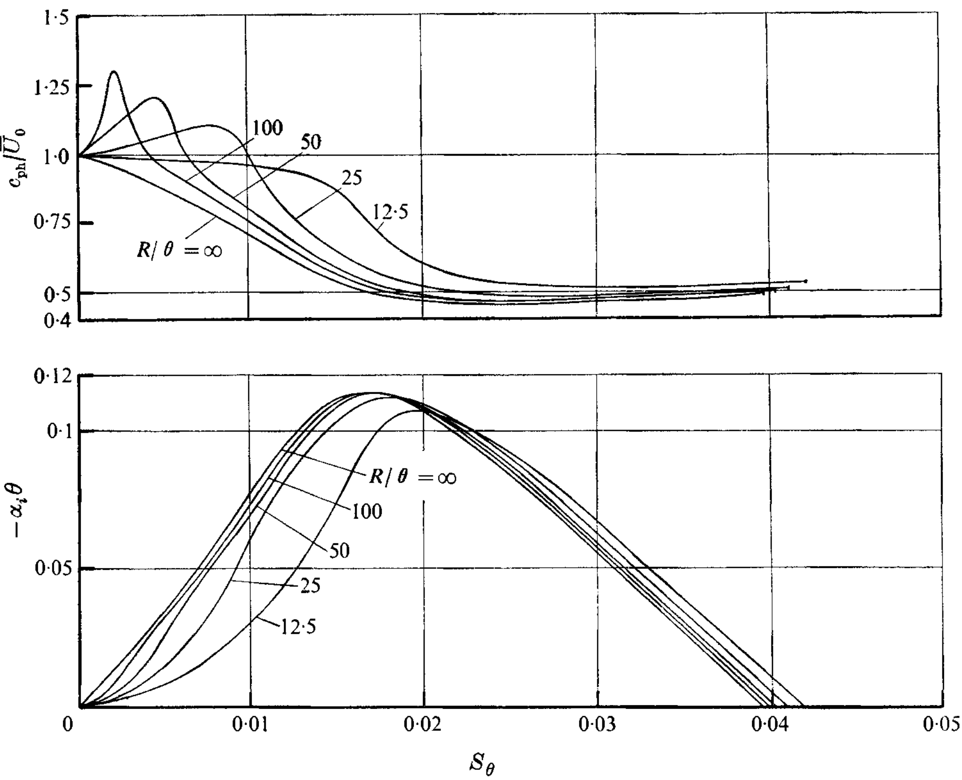

FIGURE 1. Amplification factor $-\alpha_i\theta$ and phase velocity $c_{ph}/\overline{U}_0$ as functions of the Strouhal number $S_\theta = f\theta/\overline{U}_0$. Michalke's (1971) theoretical curves.

infinitesimally thin boundary layer in their theoretical model. Indeed, the experimental data obtained by Crow & Champagne fit the theoretical results of Michalke (1971). However, the region where ultra-fast phase velocities of the jet perturbations should occur has not been investigated experimentally until now. Crow & Champagne could not find this region because they carried out their experiments with relatively thick jet boundary layers for which Michalke's computations did not predict any ultra-fast waves.

Figure 1 shows some theoretical results obtained by Michalke (1971). In this calculation the following simplifying assumptions were made. The cylindrical jet flow is parallel and inviscid. The mean velocity profile in the boundary layer of the jet has the shape of a hyperbolic tangent function. The Mach number is zero. Both the jet and the surrounding fluid at rest have the same temperature. In the curves plotted in figure 1 only perturbations with axial symmetry are considered.

The momentum thickness θ is used as a measure of the boundary-layer thickness. The ratio R/θ then indicates the relation between the radius R of the jet and the boundary-layer thickness. The jet radius R is defined in Michalke's (1971) paper as the distance from the axis to the point in the mean velocity profile where a velocity half the maximum occurs.

In the upper part of figure 1 the non-dimensional phase velocity $c_{ph}/\overline{U}_0$ is plotted *vs.* the Strouhal number $S_\theta = f\theta/\overline{U}_0$, where f is the frequency and $\overline{U}_0$ is the maximum velocity of the mean jet flow on the axis. It can be seen that ultra-fast phase velocities occur at low Strouhal numbers and for small boundary-layer thicknesses only. At sufficiently large boundary-layer thicknesses no more phase velocities exceeding the mean flow velocity can be found. The case of an isolated free shear layer is labelled $R/\theta = \infty$. Here no region of ultra-fast phase velocities can be seen. If we proceed continuously to the limit $R/\theta \rightarrow \infty$, however, we find the region of ultra-fast phase velocities at Strouhal numbers $S_\theta \rightarrow 0$. For infinitesimal boundary-layer thicknesses in a cylindrical jet, therefore, ultra-fast phase velocities still occur, at the singular point $S_\theta = 0$. It appears to follow that this phenomenon is a function of some other typical length of the problem, i.e. the jet radius. The nozzle diameter D is very close to twice the jet radius R as defined by Michalke (1971). Indeed, if one plots Michalke's results as a function of the Strouhal number $S_D = fD/\overline{U}_0$, it can be seen that the region of ultra-fast waves occurs at a Strouhal number $S_D \approx 0{\cdot}45$ (see also figure 6).

In the lower part of figure 1 the dimensionless amplification coefficient $-\alpha_i\theta$ is plotted against the Strouhal number S_θ. The r.m.s. amplitude of the velocity fluctuations is proportional to $\exp(-\alpha_i x)$, which corresponds to an exponential growth in the downstream direction x because α_i is negative. With increasing boundary-layer thickness relative to the jet radius, a decaying amplification rate at low Strouhal numbers S_θ can be seen. Again, for small Strouhal numbers the parameter S_D seems to be the more appropriate variable (see also figure 5).

In the calculations whose results are shown in figure 1 the influence of the rigid nozzle edge and of the spreading of the boundary layer were not taken into account. Therefore one should expect deviations from the theory, especially in the vicinity of the discharge edge of the nozzle and, on the other hand, downstream in the turbulent region, where the thickness of the boundary layer grows rapidly. Consequently, the present measurements were carried out at a relatively low Reynolds number Re_D (based on the nozzle diameter D and the mean nozzle exit velocity $\overline{U}_0$) at which one can expect a relatively large region of laminar flow downstream of the nozzle edge. The measurements were performed at low Strouhal numbers where one would expect ultra-fast waves according to the previous theories. This restriction to low Strouhal numbers has another advantage: that the local shape of the mean velocity profile (which changes downstream of the nozzle edge) has no significant influence on the stability behaviour, according to Michalke (1969). Since no other experimental data were available to confirm the theoretical results of Michalke and Crow & Champagne, we tried to get experimental information on what the phase velocity in this low Strouhal number region really is.

2. Experimental apparatus

The experimental set-up was identical to that used by the authors in a previous investigation (1975). The free jet was produced by means of a motor-driven radial blower with adjustable speed. The suppression of low frequency flow perturbations in the jet was given careful attention by the use of infra-sound insulation in the duct from the radial blower to the test section. The infra-sound insulation consisted of a porous plastic-foam tube. In addition, flow-induced low frequency perturbations in the free shear layer were suppressed by means of a plastic-foam rectifier in the settling chamber. The air, after passing the rectifier, was continuously accelerated, avoiding local deceleration regions at the nozzle wall. Local deceleration would possibly have produced flow separation and, as a consequence, severe and slowly varying perturbations of the free shear layer. A vortex filament nozzle, described by Michalke (1961), whose wall shape is that of the streamlines of a potential ring vortex, was used. On the axis of a vortex filament nozzle a simple expression for the relation between pressure and velocity fluctuations exists according to Bechert & Pfizenmaier (1973).

The measurements were carried out with a nozzle of diameter 30 mm in the exit plane. The mouth of the nozzle was surrounded by an annular disk having an exterior diameter three times the nozzle diameter and producing a rectangular trailing edge. Jet flow disturbances leaving the nozzle were controlled and synchronized by a loudspeaker in the settling chamber. Two interchangeable loudspeaker systems of 20 and 38 W power were used. Both systems were installed at the front end of a cylindrical cavity. The frequency responses of the systems overlapped in such a way that maximum sound pressure levels of 110–130 dB could be produced in the nozzle within a frequency range of 27 Hz–1 kHz. A reference sound pressure level was measured at an axial position 1·5 diameters upstream of the nozzle exit plane by means of a $\frac{1}{4}$ in. condenser microphone with a nose cone.

Velocity measurements were carried out with a linearized constant-temperature hot-wire anemometer which was developed by Froebel (1972) at the DFVLR-Institut für Turbulenzforschung in Berlin. Hot wires of length 2 mm adjusted perpendicular to the mean flow and tangential to the surface of the jet were used for measurements of the axial velocity fluctuations $\tilde{u}$. The probe could be moved in the radial direction either by a support driven by a synchronous electromotor or by a micrometer screw adjusted manually. By moving the hot-wire probe in the radial direction across the free shear layer, a maximum in the fluctuating axial velocity can be seen nearly in the middle of the shear layer for the low Strouhal numbers for which these measurements were made. In order to reduce the amount of data to be taken, only the maximum amplitude of the fluctuating velocity, its phase and its radial position were registered. The reader who is interested in details of the velocity fluctuation profiles is referred to the investigations by Pfizenmaier (1973) and Freymuth (1966). The hot-wire signal was filtered by a narrow-band filter, whose frequency response was calibrated individually at each frequency. The centre-frequency of the filter coincided with the frequency of the loudspeaker. The reference phase for the phase measurements was obtained from the pressure at the microphone's reference position. Phase errors due to the electronic equipment and due to the condenser microphone at low frequencies were corrected for. The reader who is interested in details of the measuring equipment is referred to a report by Bechert & Pfizenmaier (1974).

3. Choice of significant ranges of flow parameters

The general problem of an unsteady jet flow depends on the Mach number, the Strouhal number and the Reynolds number. If the Mach number is small and the characteristic length of the problem (in our case the diameter of the nozzle) is small compared with the acoustic wavelength, compressibility effects can be neglected (see, for example, Bechert 1971*a*). Owing to these restrictions the problem depends on the Strouhal number and the Reynolds number only.

We use two different Strouhal numbers: $S_D = Df/\overline{U}_0$ and $S_\theta = \theta f/\overline{U}_0$, where D is the nozzle diameter, f the frequency of the acoustic forcing field and $\overline{U}_0$ the mean exit velocity at the nozzle. In our case, where the boundary layers are thin compared with the nozzle diameter, the velocity $\overline{U}_0$ is nearly that of the inviscid flow. θ is the momentum thickness of the free shear layer, defined as

$$\theta = \int_0^\infty \frac{\overline{U}(r)}{\overline{U}_0}\left(1-\frac{\overline{U}(r)}{\overline{U}_0}\right) dr.$$

We use here the definition of the momentum thickness of a plane flow in spite of the axial symmetry of the problem because the boundary-layer thickness is small compared with the jet diameter. The momentum thickness at the nozzle edge is inversely proportional to the square root of the Reynolds number Re_D (based on the nozzle diameter) according to Michalke (1961).

Because of the coupling of θ and D via the Reynolds number, one of the two Strouhal numbers is, strictly speaking, superfluous. Nevertheless, a quantity like the position of the transition region downstream, which depends on the maximum amplification rate (see figure 1), can be described better in terms of the Strouhal number S_θ. On the other hand the phenomenon of ultra-fast jet waves, which depends mainly on the overall jet configuration, can be described better in terms of the Strouhal number S_D (see figure 6).

For physical reasons and also because of the measuring technique, the Reynolds and Strouhal numbers cannot be varied arbitrarily. In particular, the practicable Reynolds number range turns out to be very restricted. An upper boundary for our measurements with a sound-influenced free jet is $Re \approx 3\times10^5$. For Reynolds numbers exceeding this value the transition to turbulence occurs at the nozzle edge or within the boundary layer on the nozzle wall. Then phase measurements become much more complicated. However the actual Reynolds number limit is even lower and is a result of requirements of the measuring technique. The reason is that the measurements should cover a sufficient part of a hydrodynamic wavelength for the determination of the phase gradient. Experience shows that for the smallest Strouhal number of the experimental series ($S_D = 0{\cdot}275$) a length of the laminar shear layer of at least $x/D \approx 1$ should be available. This restriction can be met at a Reynolds number $Re_D \approx 10^4$ but not at higher Reynolds numbers. A measurement in the turbulent shear layer does not seem to make sense because the turbulent shear layer spreads so rapidly that no ultra-fast jet waves are to be expected.

A lower limit on the jet exit velocity $\overline{U}_0$ is given by the hot-wire anemometry. The sensitivity and dynamic frequency response of standard hot-wire anemometers in air become poor at mean velocities lower than 1 m/s (see, for example, Bechert 1971*b*). Since the peak of the alternating $\tilde{u}$ velocity component occurs near the middle of the free shear layer with a value of about $\frac{1}{2}\overline{U}_0$, a minimum mean velocity $\overline{U}_{0\,\mathrm{min}} = 5$ m/s seems to be suitable.

At a Reynolds number of $Re_D = 10^4$ and at a mean exit velocity of 5 m/s one obtains a nozzle diameter of about 30 mm. Therefore $Re_D = 10^4$ and $D = 30$ mm were chosen for all measurements. This choice of these parameters allows a comparison with previous measurements by Pfizenmaier (1973), which were carried out for the same parameter values. The Mach number was small ($M = 0{\cdot}015$) and the acoustic wavelength was large compared with the nozzle diameter at all measured Strouhal numbers. Thus the restrictions for an 'incompressible' flow were met. The Strouhal number was varied from $S_D = 0{\cdot}275$ to $S_D = 1{\cdot}77$. According to Michalke (1971) the maximum phase velocity should occur between $S_D = 0{\cdot}4$ and $S_D = 0{\cdot}5$.

4. Discussion of the measurements

In figures 2 and 3 two typical sets of results for the amplitude and phase of the $\tilde{u}$ velocity are given. Measurements were taken only at the radial position where the maximum amplitude of the r.m.s. $\tilde{u}$ velocity fluctuation occurred. This procedure was repeated at different locations in the downstream direction. The sound pressures referred to in figures 2 and 3 were taken by the reference microphone on the nozzle axis inside the settling chamber. In figures 2(*a*) and 3(*a*) a semi-logarithmic plot of the r.m.s. $\tilde{u}$ velocity fluctuation *vs.* the downstream distance from the nozzle edge can be seen. $\tilde{u}_{\mathrm{r.m.s.}}$ has been made dimensionless using the mean exit velocity $\overline{U}_0$ at the nozzle. The $\tilde{u}$ signal is filtered at the frequency of the acoustic forcing field. It can be seen that the r.m.s. $\tilde{u}$ velocity reaches a saturation amplitude whose downstream location depends on the magnitude of the sound excitation. The amplification rate (i.e. the slope of the $\tilde{u}_{\mathrm{r.m.s.}}$ curves) depends strongly on the Strouhal number and increases until the Strouhal number corresponding to the maximum amplification rate is achieved. For more information on this matter see the investigations of Freymuth (1966), Michalke (1969, 1971) and Pfizenmaier (1973).

In addition to the artificial excitation of the jet the *natural* instability was measured. These measurements were done without filtering. Thus they should not be compared directly with the measurements with artificial excitation. Nevertheless, these measurements give some information on the fluid motion which is superimposed on the artificially excited flow pattern, and the location of transition to turbulence can be roughly estimated (see also Miksad 1972). The natural excitation shows, after a region of low level which contains mainly

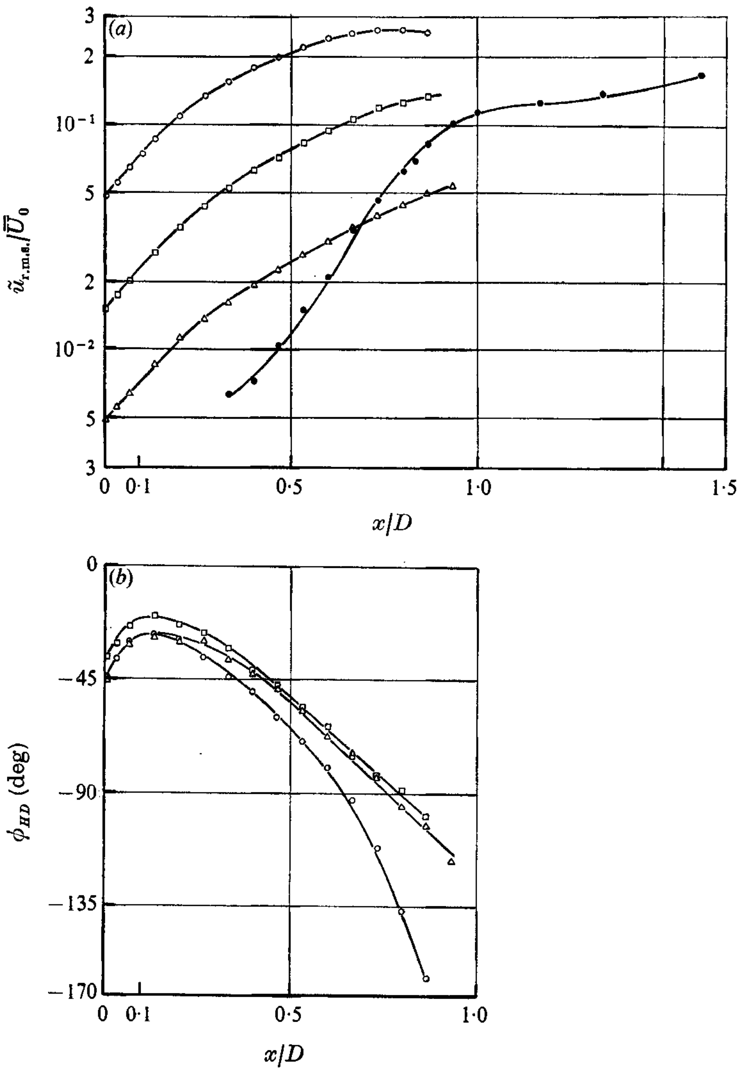

FIGURE 2. Fluctuation measurements: amplitude and phase behaviour of the sound-induced disturbances; amplitude behaviour of natural disturbances. $Re_D = 10^4, S_D = 0{\cdot}435$. ●, natural excitation; ○, 104 dB; □, 94 dB; △, 84 dB.

low frequency perturbations, an exponential growth which leads to saturation. At a short distance downstream of the saturation region turbulence occurs, and only small changes in the r.m.s. $\tilde{u}$ fluctuation amplitude subsequently take place in the streamwise direction. It is argued that the sudden spreading of the boundary layer during transition has an influence on the amplification rate of the artificially induced boundary-layer motion.

Now we return to the discussion of the artificial perturbations. Distributions of the phase in the downstream direction are plotted in parts of figures 2(*b*) and 3(*b*). A surprising situation may be noticed. First, the phase angle increases, corresponding to a phase velocity in the upstream direction, and then decreases,

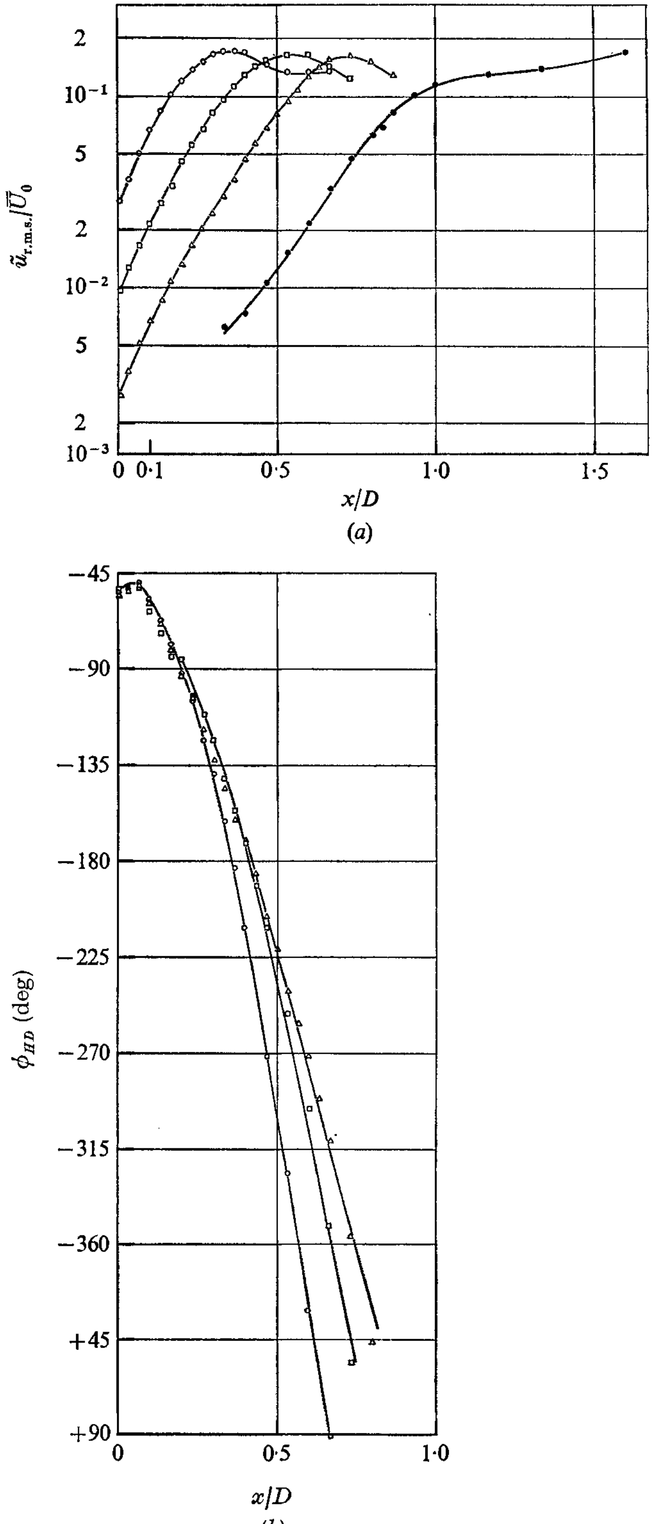

FIGURE 3. Fluctuation measurements: amplitude and phase behaviour of the sound-induced disturbances; amplitude behaviour of natural disturbances. $Re_D = 10^4, S_D = 1{\cdot}015$. Symbols as in figure 2.

as is to be expected. At a sufficiently large distance from the nozzle edge the phase gradient becomes constant if the boundary layer remains laminar within this distance (and, in consequence, the boundary-layer thickness does not change much). A constant phase gradient in the downstream direction corresponds to a constant phase velocity. The slope of the phase curves increases with increasing Strouhal number. The region of negative phase velocity near the nozzle exit becomes smaller with increasing Strouhal number, corresponding to a decreasing wavelength. Our interpretation of this phenomenon of a negative phase velocity is that it is caused by the local influence of the rigid nozzle boundary. This assumption is supported by the fact that the size of this region seems to be nearly proportional to the wavelength. According to theoretical investigations of a free shear layer leaving a semi-infinite plate (see Orszag & Crow 1970; Bechert & Michel 1975), the influence of the rigid plate on the motion of the free shear layer vanishes within some fraction of a hydrodynamic wavelength. For the determination of the phase velocity and amplification rate the regions of non-uniform phase velocity in the vicinity of the nozzle edge were excluded.

5. Consideration of linearity in the flow field

Before comparing measured results with the results of a linear theory, it is necessary to show that the flow under experimental investigation behaves linearly too. We assume that the measured $\tilde{u}_{\mathrm{r.m.s.}}$ distributions are suitable for this consideration. The beginning of nonlinearity in the fluctuating flow field can be characterized by using different conditions.

(i) The vertical distances between the measured $\tilde{u}_{\mathrm{r.m.s.}}$ curves (see figures 2*a* and 3*a*): if the alternating flow field is linear, the vertical spacing of the curves must to be equal to 10 dB (i.e. the coefficient 3·16). This is the amount of sound-pressure variance from one curve to another. The nonlinearity of the upper of two curves begins at a location where the spacing becomes smaller.

(ii) The phase behaviour (see figures 2*b* and 3*b*): the beginning of nonlinearity is apparent as growth of the gradient of a phase curve. Again this can be seen only by comparison with a phase measurement at a lower amplitude because the phase velocity obviously is not uniform in the downstream direction.

In figure 2 nonlinearity begins at $x/D \approx 0{\cdot}6$ for a reference sound pressure level of 104 dB. Obviously the nonlinear regions were excluded in the experimental determination of the phase velocity and amplification rate (for more details see Bechert & Pfizenmaier 1974).

6. Comparison between measured data and theory

If we want to determine the relative boundary-layer thickness R/θ in the experiments, we are confronted with the problem that the real flow is in fact not a parallel one. We are not in the same situation as Freymuth (1966) was when he considered a relatively small region of the flow where θ did not change significantly. In our case we have to consider a relatively large distance in the

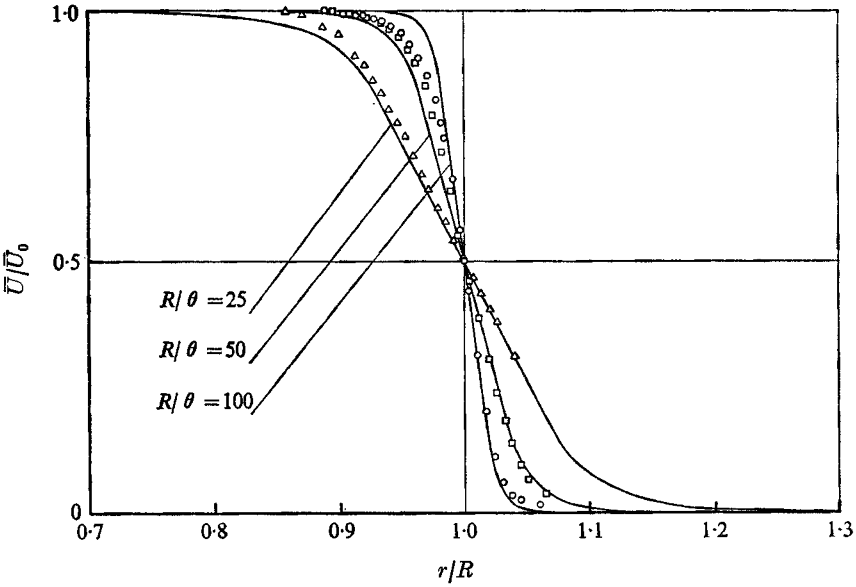

FIGURE 4. Mean velocity profiles: comparison between measured data (for $Re_D = 10^4$, $M = 0{\cdot}015$) and calculated curves ($\overline{U}/\overline{U}_0 = \frac{1}{2}[1+\tanh(\frac{1}{2}r/\theta)]$) for different values of R/θ. ○, $x/D = 0{\cdot}033$; □, $x/D = 0{\cdot}333$; △, $x/D = 1{\cdot}00$.

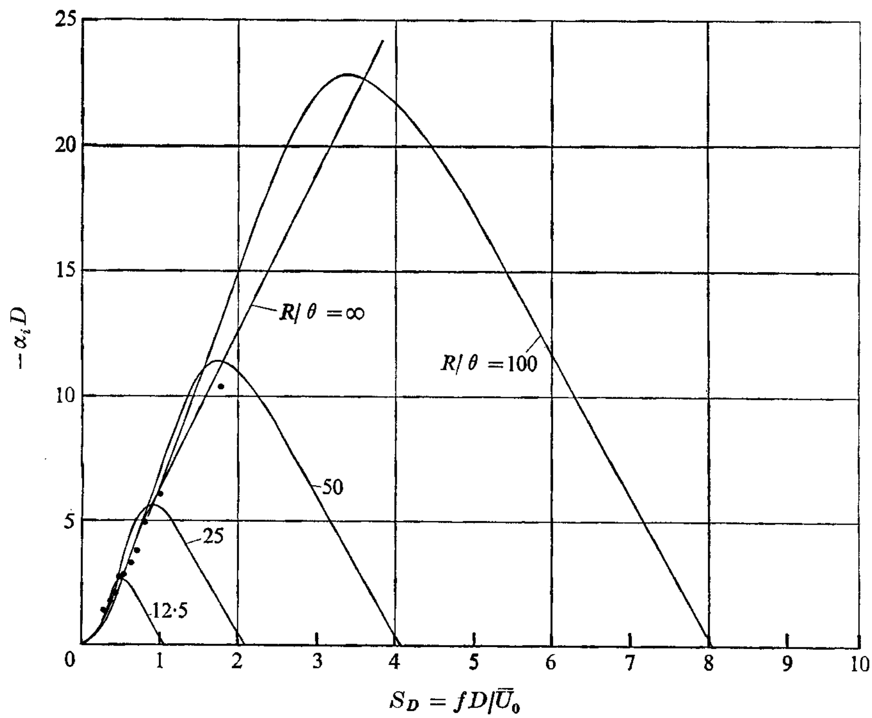

FIGURE 5. Amplification factor $-\alpha_i D$ (based on nozzle diameter) as a function of the Strouhal number S_D. ——, theoretical curves (Michalke 1971); ●, measured data for $Re_D = 10^4$, $M = 0{\cdot}015$, $R/\theta \approx 80$.

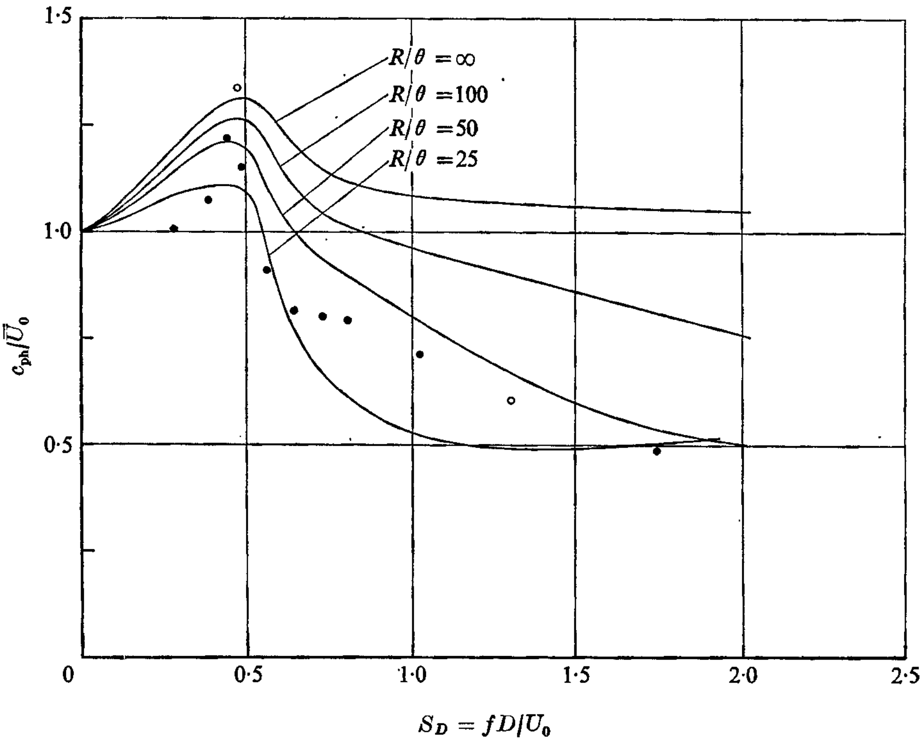

FIGURE 6. Phase velocity as a function of the Strouhal number S_D: comparison between measured data (for $Re_D = 10^4$, $M = 0{\cdot}015$, $R/\theta \approx 80$) and calculated curves (from Michalke 1971) for different values of R/θ. ●, present data; ○, data from Pfizenmaier (1973).

downstream direction, of the order of one nozzle diameter. Within this distance the mean velocity profile changes considerably (see figure 4). The relative boundary-layer thickness is $R/\theta \approx 50$ at $x/D = 0{\cdot}3$ and $R/\theta \approx 20$ at $x/D = 1{\cdot}0$. We are conscious that we are introducing a crude simplification if we extract only *one* phase velocity from the measured data in this region. Under these circumstances a comparison with theoretical curves for *different* relative boundary-layer thicknesses seems to be meaningful.

Figure 5 shows a comparison of measured and calculated amplification rates and figure 6 shows the measured and calculated phase velocities. The abscissa variable is the Strouhal number S_D, which is based on the nozzle diameter. In spite of the simplifications of the theorerical model it turns out that the agreement between theory and experiment is acceptable. Thus it has been proved experimentally that perturbations with phase velocities greater than the mean flow velocity do exist.

This investigation was carried out at the DFVLR-Institut für Turbulenzforschung, which was under the direction of the late Professor Dr -Ing. R. Wille. We wish at this point to express our gratitude to Professor Dr -Ing. A. Michalke for many stimulating discussions. Nearly all of the measuring equipment, including the free jet testing facility, has been provided by financial support from the Deutsche Forschungsgemeinschaft.

REFERENCES

BECHERT, D. 1971*a* On some simplifications concerning calculations in aeroacoustics [in German]. DLR-FB 71-25.

BECHERT, D. 1971*b* An aeroacoustic method for the calibration of hot-wire anemometers [in German]. *Proc. 7th Int. Cong. on Acoustics, Budapest*, paper 23N2.

BECHERT, D. 1972 On multiple and upstream running waves in free jets [in German]. DLR-FB 72-06.

BECHERT, D. & MICHEL, U. 1975 The control of a thin free shear layer with and without a semi-infinite plate by a pulsating flow field. *Acustica*, **33**, 5 287.

BECHERT, D. & PFIZENMAIER, E. 1973 Optical compensation measurements on the unsteady exit condition at a nozzle discharge edge. An investigation in connexion with the sound transmission through nozzles with flow [in German]. DLR-FB 73-93.

BECHERT, D. & PFIZENMAIER, E. 1974 On wavelike perturbations in a free jet running faster than the mean flow of the jet [in German]. DLR-FB 74-26.

BECHERT, D. & PFIZENMAIER, E. 1975 Optical compensation measurements on the unsteady exit condition at a nozzle discharge edge. *J. Fluid Mech.* **71**, 123.

CROW, S. C. & CHAMPAGNE, F. H. 1971 Orderly structure in jet turbulence. *J. Fluid Mech.* **48**, 547.

FREYMUTH, P. 1966 On transition in a separated laminar boundary layer. *J. Fluid Mech.* **25**, 683.

FROEBEL, E. 1972 *Instruction Manual of the DFVLR Hot-Wire Anemometer Mark* HDA III [in German]. Berlin: DFVLR.

MICHALKE, A. 1961 Theoretical and experimental investigation of the laminar boundary layer of a circular nozzle [in German]. *D. V. L. Rep.* no. 177.

MICHALKE, A. 1969 The influence of the vorticity distribution on the inviscid instability of a free shear layer [ed. Polish Acad. Sci. Warsaw]. *Fluid Dyn. Trans.* **4**, 751.

MICHALKE, A. 1970 A note on the spatial jet instability of the compressible cylindrical vortex sheet. DLR-FB 70-51.

MICHALKE, A. 1971 The instability of a compressible circular jet with finite boundary-layer thickness [in German]. *Z. Flugwiss.* **19**, 319.

MIKSAD, R. W. 1972 Experiments on the nonlinear stages of free-shear-layer transition. *J. Fluid Mech.* **56**, 695.

ORSZAG, S. A. & CROW, S. C. 1970 Instability of a vortex sheet, leaving a semi-infinite plate. *Studies in Appl. Math.* **49**, 167.

PFIZENMAIER, E. 1973 On the instability of the sound-influenced free jet [in German]. DLR-FB 73-69. (Trans. *ESRO Tech. Trans.* ESRO-TT-122.)